Alcock-Paczynski effect

(1) Assume that the universe is spatially flat and has the matter density parameter Ω_m and dark energy density parameter $\Omega_X = 1 - \Omega_m$, and that the dark energy has the equation of state of $P/\rho = \omega(z)$. Show that the comoving sizes corresponding to a redshift difference Δz and angular size $\Delta\theta$ at redshift z are given by

$$r_1 = c\Delta z/H(z) \quad \text{and} \quad r_2 = (1+z)D_A(z)\Delta\theta,$$

respectively, where the Hubble parameter $H(z) = H_0 E(z)$, $D_A = \frac{cH_0^{-1}}{1+z}\int_0^z \frac{dz}{E(z)}$, and

$$E(z)^2 = \Omega_m(1+z)^3 + (1-\Omega_m)\exp\left[3\int_0^z \frac{1+\omega(z)}{1+z}dz\right]$$

(2) Suppose there is a spherical object at redshift z subtending angular size of $\Delta\theta$ and redshift range of Δz. Plot $\Delta z/z\Delta\theta$ as a function of z.

Reference: Peebles P. J. E., 1993, Principles of Physical Cosmology, Princeton Univ. Press